Navigating the Number Fields Database at Arizona State University

When I first stumbled into algebraic number theory as a graduate student, I quickly learned that the objects we study - number fields, their rings of integers, class groups, and unit groups - are beautiful but notoriously hard to keep straight. One field might have a tiny discriminant and a trivial class group, while another with almost the same defining polynomial could behave completely differently. I remember spending hours flipping through tables in old books, trying to match a field I was working on to something already known. That is why a well-organized number fields database is such a practical tool for anyone doing serious work in this area.

Over the years I have used several such resources, but the one hosted by Arizona State University stands out for its breadth and careful curation. It is not just a list of fields; it is a structured collection that lets you search by discriminant, signature, Galois group, or class number. If you have ever needed to check whether a certain cubic field has a particular ramification pattern, you know how valuable that kind of quick lookup can be. The database saves you from recomputing what others have already done, which is the whole point of a shared computational resource.

What makes this particular number fields database useful is how it balances completeness with usability. The data goes back decades, drawing from both classic tables and modern computations. Fields are listed with their defining polynomials, discriminants, and basic invariants, but the interface also lets you drill down into more detailed information like the Galois group or the structure of the unit group. That matters because a number field is rarely just a polynomial; it is a whole ecosystem of algebraic structures, and a good database helps you see them together.

I have found it especially handy when I am exploring a family of fields and need to spot patterns. For example, if you are looking at cyclic extensions of the rationals, you can pull up all fields with a given Galois group and see how the discriminant grows. That kind of query would be tedious to do by hand, but the database makes it straightforward. The search tools are not perfect, but they are good enough that I rarely feel the need to write my own scripts for small-scale exploration.

What You Will Find Inside

The database covers number fields of various degrees, though the most extensive data is for degrees up to about 10 or 12. For each field you get:

- The defining polynomial, usually with integer coefficients and often reduced to a standard form.
- The discriminant, which is a key invariant controlling ramification and the size of the ring of integers.
- The signature, telling you how many real and complex embeddings the field has.
- The Galois group, described as a permutation group or by a standard name.
- The class number and sometimes the structure of the class group, which is crucial for understanding the arithmetic of the field.

That last item - the class number - is one of the most computationally expensive invariants to compute, so having it precomputed is a huge time saver. I have used the database to check conjectures about class numbers in families of quadratic fields, and being able to grab a few dozen examples in minutes is a luxury that earlier generations of number theorists did not have.

How It Fits Into Real Research

Let me give you a concrete example from my own work. I was studying a family of quartic fields defined by polynomials of the form x^4 + ax^2 + b, and I needed to know how often the class number was 1. Without a database, I would have had to compute each field's class group using a computer algebra system, which is doable but slow. With the number fields database, I could search for all quartic fields with a given discriminant range and check their class numbers directly. That let me focus on the interesting cases where the class number was larger than 1, which is where the real number theory lives.

Another thing I appreciate is that the database is not static. It gets updated as new computations become available, and it incorporates results from the literature. That means you are not working with stale data. Of course, no database is complete for all fields, but the coverage is good enough for most exploratory work. If you need a field that is not in the database, you can still use it to find nearby examples and get a sense of what to expect.

Limitations and Trade-Offs

No tool is perfect, and the database has its quirks. For one thing, the search interface can be a bit clunky when you want to combine multiple criteria. If you want all fields with degree 5, signature (1,2), and class number 1, you may need to run a couple of queries and merge results manually. That is not a huge deal if you are used to working with raw data, but it is something to be aware of.

Also, the database does not include everything. Higher-degree fields, especially beyond degree 12, are sparse because computing their invariants is hard. And for some fields, the Galois group or class group may be listed as unknown if the computation has not been done yet. That is honest, but it means you sometimes have to fall back on your own computations.

Still, for the vast majority of number fields that come up in everyday research - quadratic, cubic, quartic, and quintic fields, plus many sextic and septic ones - the data is reliable and well-organized. I have caught several mistakes in my own computations by checking against the database, which is something every researcher should do when possible.

Why You Should Use It

If you are a graduate student just starting in algebraic number theory, or a seasoned researcher working on a problem that involves number fields, the Arizona State University number fields database is a resource you should know about. It saves time, reduces errors, and lets you see the bigger picture of how number fields behave. It is not a replacement for understanding the theory, but it is a powerful complement that makes the theory easier to test and apply.

I still remember the feeling of flipping through pages of tables in the library, hoping to find a field that matched mine. That method worked, but it was slow and limited. Now, with a few clicks, you can access thousands of fields with their full invariants. The database is a testament to how far computational number theory has come, and it is a gift to the community that Arizona State University hosts it and keeps it accessible.

So next time you are working on a problem that involves number fields, before you fire up your computer algebra system to compute everything from scratch, check the database. It might already have what you need. And even if it does not, the examples it provides will help you understand the landscape of your problem. That is the kind of practical insight that makes a researcher more effective, and it is exactly what a good number fields database should do.